Data 140 Final Exam Reference Sheet

A. Adhikari, A. Strang

name and parameters	values	mass function or density	cdf F or survival function	expectation	variance	mgf $M(t)$
Uniform	$m \le k \le n$	$1/(n-m+1)$		$(m+n)/2$	$((n-m+1)^2-1)/12$	
Bernoulli (p)	0, 1	$p_1 = p, p_0 = q$		p	pq	$q + pe^t$
Binomial (n, p)	$0 \le k \le n$	$\binom{n}{k} p^k q^{n-k}$		np	npq	$(q+pe^t)^n$
Poisson (μ)	$k \ge 0$	$e^{-\mu}\mu^k/k!$		μ	μ	$\exp(\mu(e^t-1))$
Geometric (p)	$k \ge 1$	$q^{k-1}p$	$P(X > k) = q^k$	$1/p$	q/p^2	
"Negative binomial" (r, p)	$k \ge r$	$\binom{k-1}{r-1} p^{r-1} q^{k-r} p$		r/p	rq/p^2	
Geometric (p)	$k \ge 0$	$q^k p$	$P(X > k) = q^{k+1}$	q/p	q/p^2	
Negative binomial (r, p)	$k \ge 0$	$\binom{k+r-1}{r-1} p^{r-1} q^k p$		rq/p	rq/p^2	
Hypergeometric (N, G, n)	$0 \le g \le n$	$\binom{G}{g}\binom{B}{b}/\binom{N}{n}$		$n\frac{G}{N}$	$n\frac{G}{N} \cdot \frac{B}{N} \cdot \frac{N-n}{N-1}$	
Uniform	$x \in (a, b)$	$1/(b-a)$	$F(x) = (x-a)/(b-a)$	$(a+b)/2$	$(b-a)^2/12$	
Beta (r, s)	$x \in (0, 1)$	$\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1}(1-x)^{s-1}$	by uniform order statistics for integer r and s	$r/(r+s)$	$rs/((r+s)^2(r+s+1))$	
Exponential (λ) = Gamma $(1, \lambda)$	$x \ge 0$	$\lambda e^{-\lambda x}$	$F(x) = 1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$	
Gamma (r, λ)	$x \ge 0$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	by the Poisson process, for integer r	r/λ	r/λ^2	$(\lambda/(\lambda-t))^r,\ t < \lambda$
Chi-square (n)	$x \ge 0$	same as gamma $(n/2, 1/2)$		n	$2n$	
Normal $(0, 1)$	$x \in R$	$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	cdf: $\Phi(x)$	0	1	$\exp(t^2/2)$
Normal (μ, σ^2)	$x \in R$	$\frac{1}{\sigma}\phi((x-\mu)/\sigma)$	cdf: $\Phi((x-\mu)/\sigma)$	μ	σ^2	
Rayleigh	$x \ge 0$	$xe^{-\frac{1}{2}x^2}$	$F(x) = 1 - e^{-\frac{1}{2}x^2}$	$\sqrt{\pi/2}$	$(4-\pi)/2$	
Cauchy	$x \in R$	$1/\pi(1+x^2)$	$F(x) = \frac{1}{2} + \frac{1}{\pi} arctan(x)$			

- If $X_1, X_2, \ldots, X_n$ are i.i.d. with variance σ^2, then $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$ is an unbiased estimator of σ^2 but $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \bar{X})^2$ is not.
- For $r > 0$, the integral $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$ satisfies $\Gamma(r+1) = r\Gamma(r)$. So $\Gamma(r) = (r-1)!$ if r is an integer. Also, $\Gamma(1/2) = \sqrt{\pi}$.
- If Z_1 and Z_2 are i.i.d. standard normal then $\sqrt{Z_1^2 + Z_2^2}$ is Rayleigh.
- If Z is standard normal then $E(|Z|) = \sqrt{2/\pi}$
- The kth order statistic $U_{(k)}$ is kth smallest of $U_1, U_2, \ldots, U_n$ i.i.d. uniform $(0,1)$, so $U_{(1)}$ is min and $U_{(n)}$ is max. Density of $U_{(k)}$ is beta $(k, n-k+1)$.
- If S_n is the number of heads in n tosses of a coin whose probability of heads was chosen according to the beta (r, s) distribution, then the distribution of S_n is *beta-binomial* (r, s, n) with $P(S_n = k) = \binom{n}{k} C(r, s)/C(r+k, s+n-k)$ where $C(r, s) = \Gamma(r+s)/(\Gamma(r)\Gamma(s))$ is the constant in the beta (r, s) density.
- If $\mathbf{X}$ has mean vector $\boldsymbol{\mu}$ and covariance matrix Σ then $\mathbf{AX} + \mathbf{b}$ has mean vector $\mathbf{A}\boldsymbol{\mu} + \mathbf{b}$ and covariance matrix $\mathbf{A}\Sigma\mathbf{A}^T$.
- If $\mathbf{X}$ has the multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ, then $\mathbf{X}$ has density $f(\mathbf{x}) = \frac{1}{(\sqrt{2\pi})^n\sqrt{\det(\Sigma)}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$
- The least squares linear predictor of Y based on the $p \times 1$ vector $\mathbf{X}$ is $\hat{Y} = \mathbf{b}^T(\mathbf{X} - \boldsymbol{\mu}_\mathbf{X}) + \mu_Y$ where $\mathbf{b} = \Sigma_\mathbf{X}^{-1}\Sigma_{\mathbf{X},Y}$. Here the ith element of the $p \times 1$ vector $\Sigma_{\mathbf{X},Y}$ is $Cov(X_i, Y)$. In the case $p = 1$ this is the equation of the regression line, with slope $Cov(X, Y)/Var(X) = rSD(Y)/SD(X)$ and intercept $E(Y) - \text{slope}E(X)$.
- If $W = Y - \hat{Y}$ is the error in the least squares linear prediction, then $E(W) = 0$ and $Var(W) = Var(Y) - \Sigma_{Y,\mathbf{X}}\Sigma_\mathbf{X}^{-1}\Sigma_{\mathbf{X},Y}$. In the case $p = 1$, $Var(W) = (1 - r^2)Var(Y)$.
- If Y and $\mathbf{X}$ are multivariate normal then the formulas in the above two bullet points are the conditional expectation and conditional variance of Y given $\mathbf{X}$.
- If Y and X are standard bivariate normal with correlation r, then $Y = rX + \sqrt{1-r^2}Z$ for some standard normal Z independent of X.
- Under the multiple regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, the least squares estimate of β is $\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$.