Berkeley Stat 134 and Prob140, Spring 2017
MATH PREREQUISITES
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Here are some algebra and calculus exercises that you should be able to do with ease. I expect you to know with complete confidence that you have worked these correctly, just as you would know with complete confidence that $2 + 3 = 5$.

If you have trouble, first refer to the Appendices at the back of Pitman's text for summaries of useful results and then consult your algebra and calculus texts if you need to.

1. Consider the sequence defined by $c_i = i$, for $i = 1, 2, \ldots, 10$.

a) Find $\sum_{i=1}^{10} c_i$.

b) If possible, find $\sum_{k=1}^{10} c_k$. If this is not possible, explain why not.

2. Does the expression

$$\sum_{n=1}^{10} 2$$

make sense? If it does, what is its value?

3. Let $\{c\}$ and $\{d\}$ be sequences of real numbers so that

$$\sum_{i=1}^{100} c_i \; = \; 10 \quad \text{and} \quad \sum_{j=1}^{100} d_j \; = \; 20$$

In parts a)-c) find the value of the expression.

a) $\sum_{i=1}^{100}(4c_i + 5)$

b) $\sum_{i=1}^{100} 4c_i \; + \; 5$

c) $\sum_{i=1}^{100}(4c_i - d_i + 5)$

d) True or false:

$$\sum_{i=1}^{100}\sum_{j=1}^{100}(c_i + d_j) \quad = \quad \sum_{i=1}^{100}(c_i + d_i)$$

If you think the identity is true, find the common value of the two sides. If the identity is false, can you find the value of either of the sides?

4. Let $0 < p < 1$. Find simple expressions for

a) $\sum_{i=0}^{100} p^i$

b) $\sum_{i=0}^{\infty} p^i$

c) $\sum_{i=100}^{\infty} p^i$

5. The sum $\sum_{n=0}^{\infty} 1/n!$ can be expressed very simply. Find that simple expression and a numerical value.

6. Repeat the previous exercise for each of the sums $\sum_{i=0}^{\infty} 2^i/i!$ and $\sum_{i=0}^{\infty} 2^{3i}/i!$. If you had trouble with the previous exercise, this one might help.

7. You know that $e^0 = 1$. What we're going to need, quite often, is an approximation to e^x for a small non-zero number x. A crude approximation is 1 because x is tiny. But you can get a finer approximation by writing the first two terms in the expansion for e^x and remembering that Taylor says the rest is small compared to x.

a) Explain why $e^{0.01}$ is roughly 1.01 and $e^{-0.01}$ is roughly 0.99.

b) Use your reasoning in part (a) to explain why $\log(1+x)$ is roughly x for small x. In this class, as in much of math, log is taken to the base e.

8. How many different ways are there to arrange six people in a row?

9. A committee consists of 6 women and 4 men. How many different choices can be made if you want to select

a) a Chairperson and an Assistant Chairperson?

b) a subcommittee of two people?

c) a committee of two men and two women?

10. Let a and b be any two real numbers. You know that $(a+b)^2 = a^2 + 2ab + b^2$.

a) Analogously, write the following as a sum of four terms: $(a+b)^3$

b) Let n be a non-negative integer. Fill in the blanks:

$$(a+b)^n \;=\; \sum_{k=\_\_}^{\_\_} \_\_\_\_ a^k b^{n-k}$$

11. Calculate the following.

a) $\frac{d}{dx}\log(x^2)$

b) $\frac{d}{dx} x e^{-cx}$ where $c > 0$ is a constant

c) $\int x e^{-cx} dx$ where $c > 0$ is a constant (use part (b) or methods of integration)

d) $\int_0^\infty c e^{-cx} dx$ where $c > 0$ is a constant

12. Let $c > 0$ be a constant. $\int_0^x c e^{-cx} dx$ doesn't make sense. Why not?

13. Calculate $\int_0^1 \int_0^1 (x + xy + y) dx dy$.

14. Fill in the blanks (it really helps to draw the region of integration):

$$\int_0^1 \int_y^1 (x + xy + y) dx dy \;=\; \int_0^1 \int_{\_\_}^{\_\_} (x + xy + y) dy dx$$